Segregation of granular binary mixtures by a ratchet mechanism

Zénó Farkas,[1,2] Ferenc Szalai,[1,3] Dietrich E. Wolf,[2] and Tamás Vicsek[1]
[1]*Department of Biological Physics, Eötvös University, Pázmány P. Stny 1A, Budapest 1117, Hungary*
[2]*Department of Theoretical Physics, Gerhard-Mercator University, D-47048 Duisburg, Germany*
[3]*Computer and Automation Research Institute, Hungarian Academy of Sciences, P. O. Box 63, Budapest 1518, Hungary*
(Received 13 August 2001; published 18 January 2002)

We report on a segregation scheme for granular binary mixtures, where the segregation is performed by a ratchet mechanism realized by a vertically shaken asymmetric sawtooth-shaped base in a quasi-two-dimensional box. We have studied this system by computer simulations and found that most binary mixtures can be segregated using an appropriately chosen ratchet, even when the particles in the two components have the same size and differ only in their normal restitution coefficient or friction coefficient. These results suggest that the components of otherwise nonsegregating granular mixtures may be separated using our method.

DOI: 10.1103/PhysRevE.65.022301 PACS number(s): 45.70.Mg, 05.60.Cd, 05.40.Jc, 64.75.+g

While segregation is often an undesired effect, sometimes separating the components of a granular mixture is the ultimate goal. Since ancient times sieves have been used by humans to separate small grains from bigger ones. But nature manages to separate different kinds of grains also without sieves. Many of the segregation processes in granular matter [1–3] have recently been studied in great detail such as segregation according to particle size, shape or friction properties in a shaken box [4–6], in a rotating drum [7–10] or when poured into a thin box [11–13]. Our method for segregation, based on a ratchet mechanism, is unique because the components of the binary mixtures are collected in two buckets without any further processing, and the segregation quality can be tuned by changing the ratchet width or the load rate. Furthermore, according to our knowledge, this is the first mechanism proposed for segregating particles that only differ in hardness.

The setup we use for segregation is as follows. Particles of a binary mixture are falling into a two-dimensional box from above at a specified place. The base of the box, having an asymmetric sawtooth profile, is shaken harmonically in the vertical direction. The transport properties of homogeneous granular media in a similar setup were studied earlier [14], inspired by recent progress in the theoretical understanding of molecular motors [15,16]. In the corresponding models, known as thermal ratchets, fluctuation-driven transport phenomena can be interpreted in terms of overdamped Brownian particles moving through a periodic but asymmetric (typically sawtooth-shaped) potential in the presence of nonequilibrium fluctuating forces (such as periodic driving or switching between potentials). When a particle jumps out of the box at either the right or the left boundary, it is removed and counted, and finally the segregation quality is determined. For details on the geometry see Fig. 1. The segregation quality is $q=(2\max\{N_{1\leftarrow}+N_{2\rightarrow},N_{1\rightarrow}+N_{2\leftarrow}\}/N-1)\times 100\%$, where $N_{i\leftarrow}$ ($N_{i\rightarrow}$) denotes the number of particles of the ith component ($i=1,2$) leaving the box on the left (right) side, and N is the number of all particles in the mixture. Thus the quality of random segregation, when the particles go to the left or right side with equal probabilities is 0%. In this paper, we show results for binary mixtures in which the components contain equal numbers of particles; in other cases different quality definitions may also be appropriate.

Unlike other segregation phenomena, in which segregation is due to the collective behavior of the grains, here the interaction between the base and the individual particles is dominant, and the efficiency of segregation usually decreases with increasing number of particle-particle collisions. However, setting a sufficiently low load rate R, the quality can be high, and still "parallelizing" the procedure the separation capacity can be large (see below). In the simulation an event-driven algorithm [17] is applied with a hard-sphere collision model [18], in which the sphere-shaped particles have five parameters: mass m, radius r, normal restitution coefficient e, friction coefficient μ, and maximum tangential restitution coefficient β_0. The particles can rotate around the axis going through their center and perpendicular to the plane of the box, their moment of inertia is $\frac{2}{5}mr^2$ about their center. Since the mass of the particles does not play a role in collisions with the base but only in binary collisions, it is enough to specify that the particles have the same mass density, so the

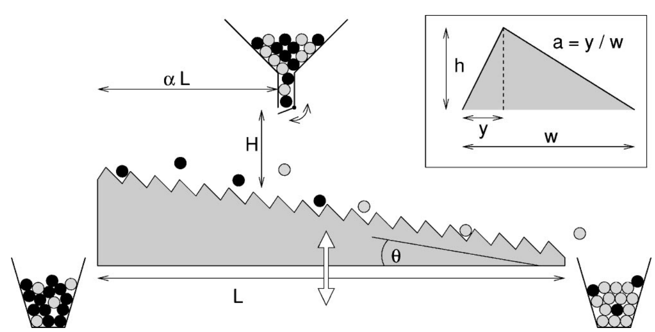

FIG. 1. Schematic drawing of the segregating setup. The particles are falling into the box of width L at αL from the left side of the box ($0<\alpha<1$) from height H with load rate R. The base is shaken sinusoidally with amplitude A and frequency f, and may be tilted by angle θ, which is positive if the left end is higher than the right one. The particles within a mixture component are exactly the same if not stated otherwise. Inset: The shape of one sawtooth is described by three parameters—width w, height h, and asymmetry parameter a.

mass is cubically proportional to the radius.

If there are only few particles in the box at the same time, then as a first approximation, the interaction between the particles can be neglected. Therefore, we investigated the motion of one particle in an infinitely wide box in detail, and found it to be chaotic [19] in most cases. A similar, but simpler model was also reported to show chaotic behavior [20]. Depending on the parameters, it is possible that the particle follows a periodic trajectory, traveling with velocity $v=fwb/c$, where b and c are integer numbers, meaning that in one period, which lasts for c vibration cycles, the particle jumps over b teeth. These periodic trajectories are not interesting for practical applications for the following reasons: (1) the transients are usually very long, and, therefore, may be more important than the asymptotic periodic trajectory for the segregation behavior; (2) periodic trajectories are not robust against collisions with other particles and other sources of noise; and, last but not least, (3) for certain conditions two periodic trajectories with opposite directions can coexist for the same type of particles. In this case one cannot predict on which side the particle tends to leave the box. We explain below how these periodic trajectories can be avoided when searching for the ratchet parameters suitable for segregating a given binary mixture. In the chaotic regime, however, the time evolution of the particle's horizontal position can be well described as drift diffusion. The connection between chaotic motion and diffusion has been investigated extensively recently [21–23]. A simple explanation for this drift-diffusion behavior can be that on time scales larger than the typical time it takes for the particle to jump to another sawtooth, the kicks of the base can be considered to be independent of each other. Furthermore, the asymmetry of the ratchet leads to an average velocity in the left or right direction. Consequently, the horizontal motion in the chaotic regime can be described statistically by two parameters: a drift velocity v and a diffusion coefficient D. An example for the drift-diffusion motion can be seen in Fig. 2.

The observation that the horizontal motion can be well approximated by drift diffusion enables us to predict segregation-related quantities [25]. First of all, the probability that a particle jumps out of the box through the right ($n_{\rightarrow}$) or the left ($n_{\leftarrow}$) boundary is: $n_{\rightarrow}(u,L,\alpha)=(1-e^{-\alpha L u})/(1-e^{-Lu})$ and $n_{\leftarrow}(u,L,\alpha)=1-n_{\rightarrow}(u,L,\alpha)$, where we introduce the notation $u=v/D$, since the probabilities depend on the drift velocity and the diffusion coefficient only through this combination. The asymptotic behavior of these probabilities in L is exponential with characteristic length $\alpha^{-1}|u|^{-1}$, which means that if the drift velocity is, say, positive and the box width is large enough, most or all of the particles arrive at the right end. The approximate explanation for this is the following: the displacement of the average particle position increases linearly in time, while the width of the probability distribution is proportional only to the square root of time (although this is exactly true only if the box width is infinite, for large box widths it is still a good approximation). Therefore, on length scales larger than $|u|^{-1}$, the drift dominates over diffusion, so that most likely the particle leaves the system at the end towards which it drifted. As a consequence, for the segregation of a binary mixture the theory suggests that if the drift velocities of the particles of the two components (v_1 and v_2) have opposite directions, then one can obtain an arbitrarily good segregation quality by choosing the box wide enough. For a fixed box width, the best segregation quality is given by $q_{\mathrm{opt}}=[n_{\leftarrow}(u_1,L,\alpha_{\mathrm{opt}})+n_{\rightarrow}(u_2,L,\alpha_{\mathrm{opt}})-1]100\%$, where $u_1<0<u_2$, and

$$\alpha_{\mathrm{opt}}=1-\frac{\ln[u_1(e^{u_2L}-1)]/[u_2(e^{u_1L}-1)]}{(u_2-u_1)L}$$

gives the optimal place of loading to obtain the best possible quality. The asymptotic behavior of q_{opt} as a function of L shows that it exponentially approaches 100% with characteristic length $(|u_1|+|u_2|)|u_1u_2|^{-1}$. Another important quantity is the *mean first-passage time*, i.e., the average time it takes for a particle to get out of the box. It is $\tau=L[n_{\rightarrow}(u,L,\alpha)-\alpha]/v$ for $v>0$. For large L values τ is linearly proportional to L, since $n_{\rightarrow}(u,L,\alpha)\rightarrow 1$ in this limit. Figure 3 shows an example for segregation of particles differing only in friction coefficient. In this example the particle parameters are: $r_1=r_2=1.5$ mm, $e_1=e_2=0.4$, $\mu_1=0.1$, $\mu_2=0.3$, and $\beta_{01}=\beta_{02}=0.4$, the ratchet parameters are: $w=8$ mm, $h=6$ mm, $a=0.4$, $\theta=-0.08$, $A=2$ mm, and $f=20\,\mathrm{s}^{-1}$, and the segregation parameters are: $\alpha=0.51$, $H=1$ cm, and $R=0.5\,\mathrm{s}^{-1}$. The corresponding diffusion parameters are $v_1=-1.58$ cm s^{-1}, $D_1=1.36\,\mathrm{cm}^2\,\mathrm{s}^{-1}$, $v_2=1.86\,\mathrm{cm\,s}^{-1}$, and $D_2=1.83\,\mathrm{cm}^2\,\mathrm{s}^{-1}$, the characteristic length is 1.85 cm. The results are obtained from the segregation of 20 000 particles, 10 000 in both components.

Since the motion of one particle in the ratchet is chaotic [20], one cannot predict the drift velocity with analytical

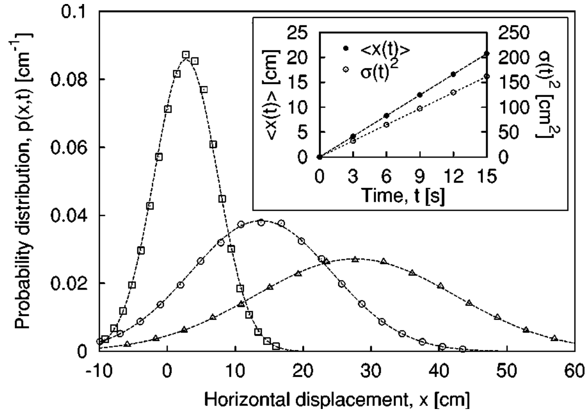

FIG. 2. Drift diffusion of a single particle in an infinitely wide system (realized by periodic boundary condition). The probability distribution of the horizontal displacement x in time $t=2$ s (square), $t=10$ s (circle), and $t=20$ s (triangle). The dashed curves show the corresponding Gaussian distributions with center vt and dispersion $\sqrt{2Dt}$ as theoretical prediction [24]. The drift velocity v and diffusion constant D are determined by line fitting: $\langle x(t)\rangle=vt$ and $\sigma(t)^2=2Dt$ (see *inset*), where $\sigma(t)^2=\langle x(t)^2\rangle-\langle x(t)\rangle^2$. The fitted values are $v=1.38\,\mathrm{cm\,s}^{-1}$ and $D=5.38\,\mathrm{cm}^2\,\mathrm{s}^{-1}$. The particle parameters are $r=1.5$ mm, $e=0.6$, $\mu=0.3$ and $\beta_0=0.4$, the ratchet parameters are $w=12$ mm, $h=8$ mm, $a=0.2$, $\theta=0$, $A=2$ mm, and $f=20\,\mathrm{s}^{-1}$.

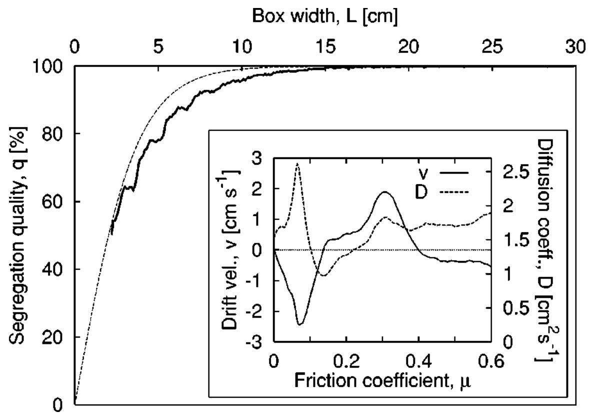

FIG. 3. Segregation of particles differing only in friction coefficient. The segregation quality (solid line) rapidly grows to 100% with increasing box width; the dashed line shows the theoretical prediction. At small box widths the deviation from the theoretical prediction (dashed line) is due to the fact that (1) the drift-diffusion approximation is valid on length scales larger than the sawtooth width, and (2) the starting state (position and velocity) of a particle is untypical in the drift-diffusion motion. Inset: The drift velocity and diffusion coefficient (measured in an infinitely wide box) as functions of the friction coefficient μ. The drift velocity changes its sign twice, at $\mu=0.14$ and at $\mu=0.4$, and is only coincidentally zero at $\mu=0$.

tools, only by computer simulation. However, a qualitative explanation for the drift reversal is as follows. The drift velocity is an average of the velocities corresponding to the chaotic trajectories. If, e.g., the parameters are such that the particle can jump over the tooth only in the left, but not in the right direction, then obviously there is a drift to the left. Beyond a critical hardness, on the other hand, the particle is able to jump over the tooth in the right direction as well. At this point a right directed component of the drift appears. It is possible, and the simulations eventually demonstrate this, that by further increasing the hardness, the right component overcomes the left component, hence the direction of the drift changes from left to right. This drift reversal may also take place if not the hardness but any other parameter is altered according to the results of the simulations.

The ratchet segregating a certain binary mixture most efficiently is sought in the following way. For both components the diffusion parameters of the one particle motion (v_1, D_1 and v_2, D_2) are measured using many different ratchets. Then we select those ratchets for which the drift velocities have opposite directions (it may happen that no such ratchet is found). Setting the box width to a reasonable value, for each of the selected ratchets the best segregation quality is predicted, and the ratchet with the highest segregation quality is chosen for segregating the mixture. However, it is possible that with this ratchet one or both of the particles have periodic trajectories, which is undesired for segregation. We describe here one possible solution to this problem: any kind of noise can destroy periodic trajectories. For example, in the computer simulation the angle of the relative velocity after a particle-base collision is changed by an amount uniformly chosen from an interval $[-\delta\phi,\delta\phi]$. We found that a rebounding angle noise $\delta\phi\approx0.05$ (measured in radian) is

TABLE I. Parameter values used to produce $3^3=27$ particle types and $5^5=3125$ ratchet types.

r (mm)	e	μ	w (mm)	h (mm)	a	f (s^{-1})	θ (rad)
1	0.4	0.1	6	6	0	16	−0.16
1.5	0.6	0.3	8	8	0.1	18	−0.08
2	0.8	0.5	10	10	0.2	20	0.0
			12	12	0.3	22	0.08
			14	14	0.4	24	0.16

enough to destroy the periodic trajectories, drastically changing the drift velocity. Hence, the diffusion parameters for both particles and each ratchet are also measured with $\delta\phi=0.05$ and 0.1. Then, for selecting a ratchet it is not enough that the drift velocities have opposite direction when $\delta\phi=0$, but their values should not change too much when the noise level is increased to 0.05 and 0.1 (we allowed a maximum relative change of 50% and 70%, respectively). The best ratchet is chosen from this restricted set by a rule that takes into account that (1) the quality should be high also in the noisy case, (2) $|v_1|$ and $|v_2|$ should be as large as possible to allow a high load rate, (3) $\alpha_{\rm opt}$ should not change much in the presence of noise, otherwise the segregation quality may decrease much due to interaction between particles. This choice of noise is not only good for avoiding periodic trajectories, but also may serve as a first approximation for taking into account the deviation of the particles' shape from a sphere. Therefore, the ratchets selected by this method are robust to some extent against deviations in shape and presumably in other parameters as well.

We checked the practicability of this procedure by finding appropriate ratchets to segregate $\binom{27}{2}=351$ different binary mixtures composed of 27 particle types, measuring the diffusion parameters for each type by simulating 3000 s of particle trajectories for 3125 ratchets. The parameters that were varied are listed in Table I, the maximum tangential restitution coefficient and the vibration amplitude were fixed $\beta_0=0.4$ and $A=2$ mm, respectively.

Then we performed a segregation simulation for each mixture with the selected ratchet (for three mixtures no proper ratchet was found in this set). We emphasize that in this segregation simulation the particles do interact with each other, hence the segregation quality depends on the load rate R (results not shown here). In practice the load rate can be increased until the segregation quality drops below the required level. The results show that in most cases good segregation quality can be achieved even if the rebounding angle noise is relatively high or the particles within the components are not uniform (see, Fig. 4).

One may think that the method presented here is not efficient for segregating real granular mixtures, since only a few grains can be present in the box at the same time, allowing only a low load rate. However, it is very easy to "parallelize" the procedure: as the box is essentially two dimensional, many boxes can be placed onto a shaking machine, and possibly the walls between the boxes can be omitted. A rough estimation shows that the capacity of such a machine would be comparable to that of the machines in use today in

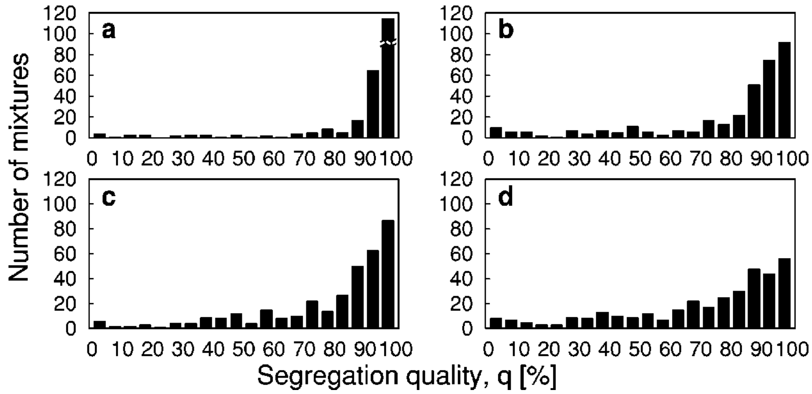

FIG. 4. Segregation quality for 351 binary mixtures. In all cases the segregation parameters are $L=30$ cm, $H=3$ cm, and $R=0.25\,\mathrm{s}^{-1}$. (a) The particles are uniform within the components, and the rebounding angle noise is $\delta\phi=0$. The number of mixtures in the last column (quality is better than 95%) is 219, and the quality is better than 70% for 320 mixtures. (b) The same as in (a) but the rebounding angle noise is $\delta\phi=0.3$. The quality is better than 70% for 270 mixtures. (c) The same as in (a) but the particles are not uniform within the components, all of the parameters (r, e, μ, and β_0) are varied by maximum 10%. The quality is better than 70% for 263 mixtures. (d) The rebounding angle noise is $\delta\phi=0.3$ and the particle parameters are varied by maximum 10%. The quality is better than 70% for 220 mixtures.

the industry for cleaning, e.g., cereal grains (in that case the capacity is in the order of 1 ton per hour). We are planning to carry out experiments to check if mixtures containing particles of the same size differing only in normal restitution coefficient or friction coefficient can be segregated by this method.

We presented a computer simulation study of a method for segregating a binary granular mixture. The segregation is performed by a ratchet mechanism, and in contrast to other segregation schemes, here not the collective behavior of the particles is dominant but the interaction between the base and the individual particles. We found that good segregation quality can be achieved even if the particles of the two components differ only in friction coefficient or hardness.

Note added in proof. Recently we learned that similar work has also been done on granular segregation by the ratchet mechanism [26].

Useful discussions with T. Fülöp and P. Tegzes are acknowledged. We thank the Computer and Automation Research Institute (Budapest) for use of their PC-cluster. Z.F. is grateful for financial support by DAAD. This research was partially supported by HSF OTKA T033104.

[1] H.M. Jaeger, and S.R. Nagel, Science **255**, 1523 (1992).
[2] *Granular Matter: An Interdisciplinary Approach*, edited by A. Mehto (Springer, New York, 1994).
[3] *Physics of Dry Granular Media*, edited by H.J. Herrmann, J.-P. Hovi, and S. Luding (Kluwer, Dordrecht, The Netherland, 1998).
[4] A. Rosato, K.J. Strandburg, F. Prinz, and R.H. Swendsen, Phys. Rev. Lett. **58**, 1038 (1987).
[5] J.B. Knight, H.M. Jaeger, and S.R. Nagel, Phys. Rev. Lett. **70**, 3728 (1993).
[6] W. Cooke, S. Warr, J.M. Huntley, and R.C. Ball, Phys. Rev. E **53**, 2812 (1996).
[7] O. Zik, D. Levine, S.G. Lipson, S. Shtrikman, and J. Stavans, Phys. Rev. Lett. **73**, 644 (1994).
[8] G. Baumann, I.M. Janosi, and D.E. Wolf, Phys. Rev. E **51**, 1879 (1995).
[9] P.- Y. Lai, L.-C. Jia, and C.K. Chan, Phys. Rev. Lett. **79**, 4994 (1997).
[10] D. Ktitarev and D.E. Wolf, Granular Matter **1**, 141 (1998).
[11] P. Meakin, Physica A **163**, 733 (1990).
[12] H.A. Makse, S. Havlin, P.R. King, and H.E. Stanley, Nature (London) **386**, 379 (1997).
[13] J. Baxter, U. Tuzun, D. Heyes, I. Hayati, and P. Fredlund, Nature (London) **391**, 136 (1998).
[14] Z. Farkas, P. Tegzes, A. Vukics, and T. Vicsek, Phys. Rev. E **60**, 7022 (1999).
[15] F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).
[16] R.D. Astumian, Science **276**, 917 (1997).
[17] B.D. Lubachevsky, J. Comput. Phys. **94**, 255 (1991).
[18] O.R. Walton in *Particulate Two-Phase Flow* edited by O.M. Roco, (Butterworth-Heinemann, Boston, 1992).
[19] G. L. Baker and J. P. Gollup, *Chaotic Dynamics: An Introduction* (Cambridge University Press, Cambridge, England, 1990).
[20] J. Duran, Europhys. Lett. **17**, 679 (1992).
[21] P. Gaspard and G. Nicolis, Phys. Rev. Lett. **65**, 1693 (1990).
[22] P. Gaspard *et al.*, Nature (London) **394**, 865 (1998).
[23] C.P. Dettmann and E.G.D. Cohen, e-print nlin.CD/0001062.
[24] H. Risken, *The Fokker-Planck Equation* (Springer, Berlin, 1989).
[25] Z. Farkas and T. Fülöp, J. Phys. A **34**, 3191 (2001).
[26] M. Levanon and D.C. Rapaport, Phys. Rev. E **64**, 061304 (2001).